Ch.12, Sec.2 - Rogawski & Adams' Calculus

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Ex. 2.60

67. A median of a triangle is a segment joining a vertex to the midpoint of the opposite side. Referring to Figure 20(A), prove that three medians of triangle ABC intersect at the terminal point P of the vector $\frac{1}{3}(\mathbf{u}+\mathbf{v}+\mathbf{w})$. The point P is the *centroid* of the triangle. *Hint:* Show, by parametrizing the segment $\overline{AA'}$, that P lies two-thirds of the way from A to A'. It will follow similarly that P lies on the other two medians.

$$
\begin{aligned}
L &= P_0 + tv \\
&= (x_0, y_0, z_0) + t(a, b, c)
\end{aligned}
$$

$$
\begin{aligned}
x = x_0 + ta &\rightarrow t = \frac{x - x_0}{a} \\
y = y_0 + tb &\rightarrow t = \frac{y - y_0}{b} \\
z = z_0 + tc &\rightarrow t = \frac{z - z_0}{c}
\end{aligned}
$$

Ex. 2.66

66. Show that the line in the plane through (x_0, y_0) of slope m has symmetric equations

$$x - x_0 = \frac{y - y_0}{m}$$

Be definition of a slope, We have two points on the line: (x_0, y_0) and $(x_0 + 1, y_0 + m)$. By page 640, We take the directional vector: $v = (x_0 + 1, y_0 + m) - (x_0, y_0) = (1, m)$. Thus,

$$
\begin{aligned}
x = x_0 + (1)t &\rightarrow t = x - x_0 \\
y = y_0 + (m)t &\rightarrow t = \frac{y - y_0}{m}
\end{aligned}
$$

Hence, The symmetric form is satisfied.

Ex. 2.67

60. Let $\mathcal{L}$ be the line through $P_0 = (x_0, y_0, z_0)$ with direction vector $\mathbf{v} = \langle a, b, c \rangle$. Show that $\mathcal{L}$ is defined by the symmetric equations (10). *Hint:* Use the vector parametrization to show that every point on $\mathcal{L}$ satisfies (10).

$$\begin{aligned} \vec{OA} &= v \\ \vec{OA'} &= \frac{1}{2}(w-u)+u \\ &= \frac{1}{2}(w+u) \\ \vec{OA'} - \vec{OA} &= \frac{1}{2}w + \frac{1}{2}u - v \end{aligned}$$

Taking 2/3 of it: $\frac{2}{3}(\vec{OA'} - \vec{OA}) = \frac{1}{3}w + \frac{1}{3}u - \frac{2}{3}v$.

P is the terminal of vector: $v + \frac{1}{3}w + \frac{1}{3}u - \frac{2}{3}v = \frac{1}{3}w + \frac{1}{3}u + \frac{1}{3}v$.

Symmetrically, Taking $\frac{2}{3}$ of segments BB' and CC' yields the vector $\frac{1}{3}w + \frac{1}{3}u + \frac{1}{3}v$, and thus point P lies on the other two medians as well.