Chapter 05

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Contents

Problems

1

a

$$\alpha^{-1} = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 3 & 5 & 4 & 6 \end{bmatrix}$$

b

$$\beta\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 6 & 2 & 3 & 4 & 5 \end{bmatrix}$$

c

$$\alpha\beta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 2 & 1 & 5 & 3 & 4 \end{bmatrix}$$

3(b)

b. (124)(35)(6)

6(a)

a. disjoint cycles form: (12)(356). Order is 6.

8(c,d)

c. (17)(16)(15)(13).

d. (24)(23)(15).

10

We want to find some permutation

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ & & & & & \end{bmatrix}$$

where:

- Order is 15, i.e *lcm* of disjoint cycles lengths is 15, and
- Even, i.e Has an even number of 2-cycles.

Observe $15 = 3 \cdot 5$ which suggests two disjoint cycles of lengths 3 and 5. A standard candidate is (123)(45678). Its 2-cycles form: (48)(47)(46)(45)(13)(12), A total of even 6 cycles.

The permutation in matrix form is:

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 3 & 1 & 5 & 6 & 7 & 8 & 4 \end{bmatrix}$$

17

Using *theorem 3.3* (page 64), It suffices to show the set of even permutations are closed under permutation composition. By definition, Given even permutations $\alpha = (ab)(cd)\dots(ef)$ and $\beta = (gh)(ij)\dots(kl)$, The composition $\alpha\beta = (ab)\dots(kl)$ consists of even number of 2-cycles, As even + even = even.

23

Let H be a subgroup of S_n. Assume not every member is even. We show H must have an equal number of even and odd members.

We follow the same proof approach of *theorem 5.7* (page 104). We know there is an odd member α. For every odd β, $\alpha\beta$ is even, So there as many evens as there are odds. For every even β, $\alpha\beta$ is odd, So there are as many odds as there are evens. Therefore, the number of even and odd members are equal.

25

The identity permutation is even. Not closed as the composition of two odd members is even.

43

For $n \geq 3$, It is easy to see $(12) \in S_n$ and $(23) \in S_n$. However

$$(23)(12) = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & \dots \\ 3 & 1 & 2 & 4 & 5 & \dots \end{bmatrix} \neq \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & \dots \\ 2 & 3 & 1 & 4 & 5 & \dots \end{bmatrix} = (12)(23)$$